

# Static Generalized Brans-Dicke Universe and Gravitational Waves Amplification

Marcelo.S.Berman<sup>(1)</sup>

and Luis.A.Trevisan<sup>(2)</sup>

(1) Instituto de Tecnologia do Paraná-  
-Tecpar-Grupo de Projetos Especiais.

R.Prof Algacyr M. Mader 3775–CIC-CEP 81350-010  
Curitiba-PR-Brasil

Email: marsambe@tecpar.br

(2) Universidade Estadual de Ponta Grossa,  
Demat, CEP 84010-330, Ponta Grossa,Pr,

Brasil email: latrevis@uepg.br

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## Abstract

We present a Static Universe in a generalized Brans-Dicke gravity theory, where the coupling “constant” varies with time, as well as the scalar field. There is amplification of gravitational waves in such Universe, at least when it is “young”.

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# 1 Introduction

The first models of the Universe that we can find in the literature are Einstein's static ones (see, for instance, Tolman [1]).

Today, the study of static models has a two-fold aim. First, it throws light on the theory, and one can compare with expansion models and conclude which physical effects are due on the expanding properties of scale-factors, and which are not. Second, we would like to remind that, according to some critical approaches, it is to be questioned, at least just for its own sake, whether there could be an explanation to "cosmological " red-shifts, other than a recessional effect.

With these objectives in mind, Berman, Som and Gomide studied static Brans-Dicke models[2]; Berman [3] studied their stability properties, too. Berman also studied [4] such Universes in "modified " B.D. theory, and a static Universe with a magnetic field in Einstein Cartan's Cosmology [5]. Afterwards, Berman and Som [6] also delved into the static solutions in Wesson's 5D theory of gravity. Earlier, Berman [7] also studied inhomogeneous static models in B.D theory.

All these studies reveal details of theoretical characteristics not necessarily associated with evolutionary models. We shall now study a static model with time varying coupling "constant", as well as a variable scalar field. Scalar tensor cosmologies were earlier studied by Barrow [8]. We wish to point out that the core of the theory on scalar-tensor cosmologies was undertaken by J.D.Barrow and collaborators(see for instance references [9][11]).

# 2 The Field Equations

Let us consider the following action:

$$L_{\Phi} = -\Phi R + \Phi^{-1}w(\Phi)\partial_a\Phi\partial^a\Phi + 16\pi L_m \quad (1)$$

where  $L_m$  is the Lagrangian for matter fields, and  $\Phi$  is the scalar field. If  $w = const$  we obtain the Brans-Dicke theory [10]. This Lagrangian was adopted by Barrow and Maeda [9] . For a discussion about the Lagrangians of the scalar theories of gravitation, see [12].

We now shall undertake the study of such Universes in an  $\omega = \omega(\phi)$  Brans-Dicke generalized theory, the field equations being:

$$G_{ab} = -\frac{8\pi}{\phi}T_{ab} - \frac{\omega}{\phi^2} \left[ \phi_a \phi_b - \frac{1}{2}g_{ab}\phi_i\phi^i \right] - \frac{1}{\phi} [\phi_{a;b} - g_{ab}\square\phi] \quad (2)$$

and

$$[3 + 2\omega] \square\phi = 8\pi T - \left( \frac{d\omega}{d\phi} \right) \phi_i\phi^i \quad (3)$$

while we must impose conservation of energy-momentum tensor:

$$T_{;b}^{ab} = 0 \quad (4)$$

For a static metric, and a flat Universe,

$$ds^2 = dt^2 - R_0^2 [dx^2 + dy^2 + dz^2] \quad (5)$$

we find, then:

$$\frac{8\pi\rho_0}{\phi} + \frac{\omega}{6} \frac{\dot{\phi}}{\phi^2} = 0 \quad (6)$$

$$\ddot{\phi} = -\frac{\dot{\phi}\dot{\omega}}{3+2\omega} + \frac{8\pi}{3+2\omega} (\rho_0 - 3p_0) \quad (7)$$

where  $\rho = \rho_0 = \text{const.}$

For a perfect gas equation of state

$$p = \alpha\rho = p_0 = \text{const.} \quad (8)$$

Let us suppose that:  $\ddot{\phi} = 0$  and  $\omega \neq -3/2$ . We find the solution :

$$\omega = 8\pi (\rho_0 - 3p_0) t \quad (9)$$

and

$$\phi = \frac{6\rho_0}{3p_0 - \rho_0} t \quad (10)$$

The solution is time-dependent, although the scale-factor is constant, like the energy density and pressure. If  $\rho_0 > 3p_0$ , we have  $\omega \geq 0$ , and:

$$\lim_{t \rightarrow \infty} \omega = \infty, \quad (11)$$

which means that General Relativity is the limiting gravity theory, for an “old ” Universe.

It is clear that the time variation of  $\omega$  and  $\phi$  cannot be attributed to an evolution in the scale-factor.

### 3 Gravitational Waves.

Barrow et al. [11] calculated the amplification equation for gravitational waves, which has here the following form:

$$\ddot{Y}_k + \frac{\dot{\phi}}{\phi} \dot{Y}_k + \left[ \frac{k^2}{R_0^2} - \frac{\dot{\omega}}{2\omega + 3} \frac{\dot{\phi}}{\phi} \right] Y_k = 0 \quad (12)$$

where  $k = |\vec{k}| = 2\pi R_0/\lambda$ .

For an “old ” Universe, we would have a non-amplified harmonic function, while for a young Universe, we are left with the equation:

$$\ddot{Y}_k + t^{-1} \dot{Y}_k - \frac{1}{2} t^{-2} Y_k \cong 0 \quad (13)$$

which has the solution

$$Y_k = A t^{\frac{\sqrt{2}}{2}} \quad (14)$$

( $A = const$ ), and then we have amplification of gravitational waves.

### 4 Conclusion

We studied a Static generalized Brans-Dicke theory. It is shown that the time variation of the scalar field  $\phi$  and of the coupling “constant ”  $\omega$  are not a consequence of the variation of the scale-factor, which is constant, in this work, by hypothesis. Another effect of such model is the amplification of gravitational waves in the early Universe.

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